Senior Thesis

Derivation of three methods of calculation of heat conduction, applications of these methods, and calculation of time from emplacement to solidification of dikes of varying widths, Egersund dike swarm, SW Norway

Presented in partial fulfillment of the requirements for the degree of Bachelor of Science in the Department of Geological Sciences

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2003

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Introduction

In this paper, several methods for solving problems of heat flow will be shown. A number of examples will be worked out in order to show the practicality of these methods. Finally, the methods will be applied to real data to confirm their practicality and validity.

A quantitative description of heat being transferred through a medium, or from one medium to another, is not a trivial matter. The mathematics involved can be rather complicated. There have been several methods devised for describing and quantifying heat flow. The methods that are described in this paper are an analytical calculation of heat conduction using Fourier's method, a method of numerical analysis, and the Crank-Nicolson finite difference technique. Each of these methods is useful and has its strengths in practicality and precision.

A number of theoretical problems are devised to show the strategies taken in applying these methods of calculation to circumstances in the natural world. The following will explore the problems of calculating a geothermal gradient, calculating the time required for partial melting to progress a certain distance through country rock using Fourier analysis and numerical analysis, and calculating the temperature gradients in a cooling lava lake using Fourier's method, numerical analysis, and the Crank-Nicolson finite difference technique.

Once these techniques are shown to produce valid results, the geometry can be slightly changed inorder to apply the analysis to cooling dikes of varying widths. In the final part of this paper, the diffusion equation which is derived from Fourier's equation is used to calculate the time from emplacement to solidification for a series of dikes known as the Egersund Dikes which are located in southwest Norway. The resulting times can then be compared to results of calculations made using different properties, data, and methods entirely.

Methods

Heat transfer can be a complicated subject, which involves considerable mathematics. Only specific derivations will be used here, which are helpful for solving the problem of dike cooling. The derivations can be somewhat complicated, but the results are easily used and helpful for a variety of geologic problems.

Heat can be transferred in three different ways. The first method, conduction, involves the transference of kinetic energy from one atom or molecule to the next in order to transmit thermal energy. The second method of heat transfer is convection. This is the process of physically moving heated material from hotter to cooler regions within a system. The third method is radiation. Here, energy is transferred directly from one place to another by electromagnetic radiation (Philpotts 1990).

The methods described in this paper will be dealing only with conduction. This is the most important factor controlling the cooling of most igneous bodies and especially dikes. Convection will play a more important role, generally, in larger bodies of magma. Radiation, also, is generally insignificant in the applications discussed here. Diffusion is the mechanism of heat conduction and it is therefore described by the diffusion equation, which will be described and derived. The following is a brief discussion of the derivation of the equations that are used for these conduction calculations as described by Philpotts (1990).

General Theory

The most basic model for heat conduction is that of two parallel plates where one plate is cooler than the other, and therefore heat is transferred. If the difference in temperature is ΔT, and the plates are separated by a distance Δx, then the amount of heat Q that is transferred from one plate to the other over area A is given by

$$Q = - KA({}^{\Delta Tt}/_{\Delta x}) \qquad (1)$$

where Q is directly proportional to the difference in temperature and the time during which heat flows, and inversely proportional to the distance between the plates. The constant of proportionality K is the thermal conductivity.

When the plates are moved closer together, the limiting factor dT/dx is the thermal gradient at that point. If the thermal gradient is taken in the direction of heat flow, it will be negative. The rate of heat flow is a positive number and can be found by differentiating equation (1),

$$^{dQ}/_{dt} = - KA({}^{dT}/_{dx}) \qquad (2)$$

If this rate of heat flow is taken across a unit area it is known as the heat flux and is given by,

$$J_x = {}^{dQ}/_{dtA} = - K({}^{dT}/_{dx}) \qquad (3)$$

This is known as *Fourier's Law.*

Now, using thermal conductivity, we know the rate at which heat can be transferred into, or out of a substance. However, it is not useful, practically, to know the measure of heat flow. It is more useful in geologic problems to know how much the temperature of the material changes as a result of this flow of heat.

In order to find the temperature change it is necessary to use the factor of the heat capacity of the material. Heat capacity C_p is defined as the amount of heat necessary to raise the temperature of a unit mass of a substance by one degree at a constant pressure. The amount of heat needed to raise the temperature of a unit volume of the substance by one degree is $C_p\rho$ where ρ is the density of the material.

Now, if K which is the quantity of heat that flows in unit time through unit area of a plate of unit thickness with unit temperature difference between its faces is divided by $C_p\rho$ which is the amount of heat that must flow into the unit volume for its temperature to be raised one degree the result is the change of temperature produced in the unit volume by the quantity of heat that flows.

$$k = K/ C_p\rho \qquad (4)$$

This is known as the thermal diffusivity and is a property of material that is needed in order to determine the change in temperature that is caused by an influx of heat. Most rocks and magmas have a thermal diffusivity of $10^{-6} m^2/s$.

Conduction Across a Plane Contact

The first model to consider after the theoretical model of the two plates is that of heat being conducted across a plane contact. This has useful geological applications such as the contact between any magmatic body and the surrounding country rock, or even the contact between magma and air. It is assumed here that the contact is nearly planar.

The boundary conditions must first be taken into account. The magma is initially emplaced with a temperature of T_0. The country rock is usually taken to be 0^oC. This is generally close enough to the true temperature so that the calculations are not effected drastically and makes the calculations easier. However, in order to use the actual country rock temperature, the temperature scale only needs to be adjusted up or down accordingly. The point where the temperature of the country rock remains constant moves a greater distance away from the contact after intrusion. Likewise, the intrusion is wide enough that magma remains inside the body at the initial temperature T_0 and the distance from the contact to the area with this temperature increases with time. Eventually, with time, no magma with the initial temperature will remain. This amount of time depends on the size of the intrusion. At this time the boundary conditions will change and so will the approach to the problem. The distance from the contact in either direction will be denoted as x. Negative values of x will be into the country rock, and positive is in the intrusive body. So, initially T/T_0 will be 1 for $x>0$, and 0 for $x<0$. As time progresses, T/T_0 will continue to tend toward these values.

In order to determine the temperature T, at a distance x, at time t after intrusion, a derivation of Fourier's conduction equation (3) is used. This derivation is rather involved and is worked out by Carslaw and Jaeger (1959). The solution assumes that the thermal diffusivity *k* of the magma and the country rock are the same. This is a reasonable assumption and the solution given is

$$T/T_0 = \tfrac{1}{2} + \tfrac{1}{2}\,\mathrm{erf}\left({}^{x}/_{2\sqrt{kt}}\right) \qquad (5)$$

Here erf represents the error function. Values for the error function can be found in tables of many texts and computed by many computer programs. Philpotts (1990) gives a set of equations that can be used to estimate error function values if a table or recent computational software is not available.

It is seen by this equation that the temperature of the country rock at the contact (i.e. x = 0) can never be more than ½ of the initial temperature T_0 of the magma as long as the boundary conditions do not change.

Carslaw and Jaeger (1959) give a solution for the temperature through a sheet like intrusion with thickness 2a (half-width of a) where distance *x* is measured from the center of the body.

$$T/T_0 = \tfrac{1}{2}\left[\mathrm{erf}\left({}^{a-x}/_{2\sqrt{kt}}\right) + \mathrm{erf}\left({}^{a+x}/_{2\sqrt{kt}}\right)\right] \qquad (6)$$

This equation is the diffusion equation, and can be applied to a variety of problems, such as changes in composition of exsolution lamellae, by changing the coefficient in order to solve for a different property such as chemical composition in this case.

Numerical Analysis

Sometimes, calculations of heat transfer cannot be carried out in the ways described above. Many heat flow problems are too complicated and do not have an exact solution that can be found analytically. Problems such as these need to be solved using a numerical analysis. In this method, the material of interest is broken down into small volumes or cells. The heat flow through each of these cells can be solved using Fourier analysis. In this method though, the volumes are not taken to infinitely small sizes in order to arrive at a differential equation. The size of each cell remains finite. The heat flow through each cell is in the end summed to find the heat transfer through the body. In most cases, sufficiently accurate results can be found with a reasonably low number of cells. Of course though, the more cells that are used at smaller sizes, the closer the result will be to the exact solution of the differential equation.

The heat flow through a body is assumed to flow in only one direction due to the extent of the magma body. For example, this direction would be horizontally from the center to the outside surface in a vertical dike, straight out into the country rock from a large batholith, or straight up in the z direction toward the surface in a lava lake. The heat that flows through each cell is designated Q_i. The physical properties are included for each cell. These include thermal conductivity K_i, thermal diffusivity k_i, heat capacity C_i, density ρ_i, and volume V_i.

According to Fourier's law (1), the quantity of heat that flows from cell (i+1) to cell i is

$$Q_{i+1} = -[K(T_i - T_{i+1})A\Delta t]/\Delta z \qquad (7)$$

where Δt is the time involved.

The change in heat content of any cell is equal to the amount of heat that comes into the cell minus the amount of heat that flows out of the cell, plus any change in heat that is caused by outside sources or sinks Q_i^*. Sources may include heat of crystallization, and sinks could include heat used in evaporation of water. This relationship holds due to the conservation of thermal energy. The change in heat content of a cell is then shown as

$$\Delta Q_i = (Q_{i+1} - Q_i) + Q_i^* \qquad (8)$$

Now the change in temperature is needed to make things useful. The heat capacity per cell will be $C_i \rho_i V_i$. When this is divided by the change in heat content ΔQ, we get the change in temperature of the cell ΔT_i. Then the change in temperature is expressed as

$$\Delta T_i = \Delta Q_i / C_i \rho_i V_i = [(Q_{i+1} - Q_i) + Q_i^*] / C_i \rho_i V_i \qquad (9)$$

Crank-Nicolson Finite Difference Technique

Another method of solving heat flow problems is the finite difference technique, which was developed by Crank and Nicolson in 1947. The technique has many applications including problems of heat sources and sinks, and problems considering movement of the mass under consideration (Philpotts 1990). Here, only the one directional heat conduction application will be explored.

Fourier's equation for the conduction of heat in one direction can be written as

$$dT/dt = k(d^2T/dz^2) \tag{10}$$

This equation shows that the temperature is a function of both time and distance. What the Crank-Nicolson finite difference method does is to put these two variables on a grid. On this grid, the number of increments of dz and dt are represented by m and n, respectively. Given a temperature at any point on this grid, such as $T_{m,n}$, the temperature at any adjoining point on the grid such as $T_{m+1,n}$ can be shown by a Taylor series expansion. The points are relatively close, so only the first few terms of the expansion are needed. These expansions for several points that are near the given point, along with some variations of Fourier's equation, and some further manipulation, which is given in Philpotts (1990), leads to

$$T_{m+1,n} = \tfrac{1}{4}(T_{m+1,n} + T_{m-1,n} + T_{m+1,n+1} + T_{m-1,n+1}) \tag{11}$$

This equation gives the temperature at a point in terms of four temperatures that are at points that differ from it by one finite difference in space and one finite difference in time. Use of this equation requires and establishment of boundary conditions. The values of dt and dz must be set so that $kdt/(dz)^2 = 1$.

Applications

Geothermal Gradient

The first problem that these methods of calculation can be applied to is that of constructing a geotherm. This is a fairly simple, but useful problem and application. As depth into the crust increases, so does the temperature of the rocks. This is due to the heat that is being conducted out of the Earth's center. In order to determine how temperature will increase with depth, calculations must be made to find the amount of heat that is conducted through each layer of the Earth. This increase in temperature with depth is not linear and not constant.

The following is an example that works out problem 1-4 from Philpotts. We start with a model of a multi-layered Earth. In this example there are assumed to be three layers of different thickness in the continental lithosphere. The surface heat flow is given as 46 mW/m^2. The three layers in this problem are given the following properties.

Layer	Thickness (km)	A ($\mu W/m^{-3}$)	K (W/mK)
1	10	2.1	2.51
2	30	0.26	2.51
3	60	0.0	3.35

In order to solve this problem we need to work backwards. The heat flux coming into the bottom of the first layer is that which is coming out of the top of the second layer and the heat flux coming into the bottom of the second layer is that which is coming out of the top of the third layer. Knowing the surface heat flux Ht, the heat flux at the bottom of layer two Hb is found by

$$(Ht - A)*z \qquad (12)$$

$$(0.046\ W/m^2 - 2.1*10^{-6}\ W/m^2)*10000 = 0.025\ W/m^2$$

Now, knowing the heat flow through the bottom of the first layer, or in other words, through the top of the second layer, the heat flow through the bottom of the second layer, or the top of the third layer can be calculated in the same way.

$$(0.025\ W/m^2 - 0.26*10^{-6}\ W/m^2)*30000 = 0.017\ W/m^2$$

Now the geothermal gradient can be calculated through each of these layers using the Taylor series expansion. The temperature at the surface of the earth is taken to be $0^{o}C$. The temperature T_z at any depth z in the first layer can be given by

$$T_z = 0 + (0.046/K) * (z\text{-}0) * 1000 - (A/2K) * ((z\text{-}0) * 1000)^2 \qquad (13)$$

The temperature at z = 10 km, or the base of the first layer is found to be $141.434^{o}C$. This is now used as the temperature at the surface of the second layer to determine the geothermal gradient through the second layer in the same way.

$$T_z = 141.434 + (0.025/K) * (z-10) * 1000 - (A/2K) * ((z-10) * 1000)^2 \quad (14)$$

The temperature at the base of layer two (z = 40 km) is found to be 393.625. The third layer is taken to be the bottom of the region in this problem, so there will be no heat coming in from below it. Because of this, the Taylor series expansion reduces to only the first two terms and is linear.

$$T_z = 393.625 + (0.017/K) * (z - 40) * 1000 \qquad (15)$$

The geothermal gradient changes for each layer because of the amount of heat that is added into it from the layer below. Because the temperature is the same at the top of one layer and the bottom of the layer below it, the traces of the gradients can be pieced together into one curve. This is shown in figure 1. The resulting temperatures are given with respective depths in appendix A.

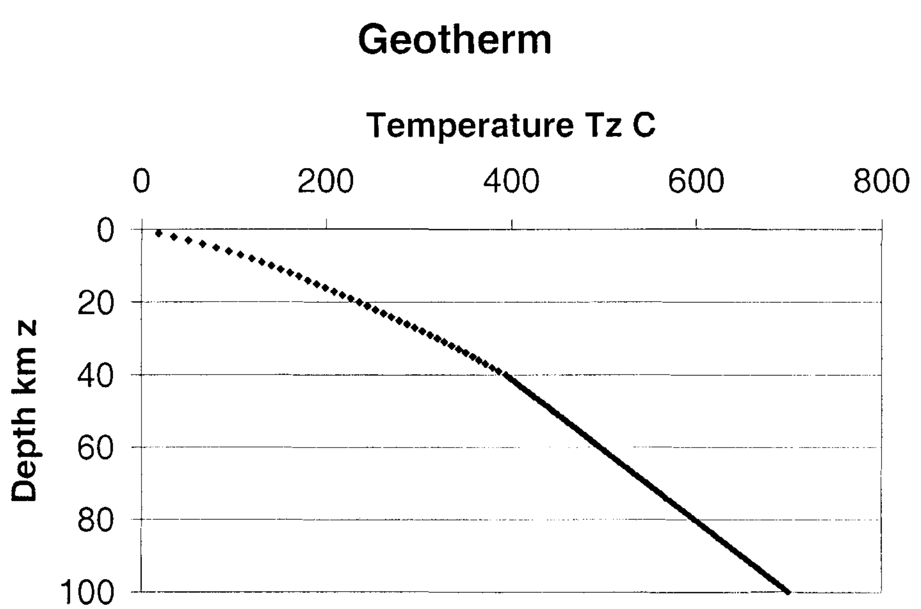

figure 1

Partial Melting Calculation

Continuous contact of magma with country rocks can cause partial melting in the country rocks. When this is the case, the heat flow will occur in such a manner as described in the equations above. The given parameters of this problem are from problem number 5-7 of Philpotts. The contact temperature is given to be 1000°C. The country rocks are at a temperature of 100°C prior to intrusion of the magma. The country rocks will begin melting at 700°C. The thermal diffusivity, as usual, is assumed to be $10^{-6}m^2/s$. Latent heat of crystallization is ignored in this problem, as it will be in the following problems.

The problem then is to determine how long it will take under these conditions for partial melting to occur 2m from the contact into the country rock. The temperature of the country rock is adjusted from 100°C to 0°C and then the partial melting temperature is adjusted the same amount from 700°C to 600°C. Then, the contact temperature is also adjusted from 1000°C to 900°C. The initial temperature T_0 is then assumed to be twice this amount or 1800°C because the contact temperature remains constant throughout the duration of the problem. Now, we use equation (5) to solve the problem. First we find that

$$T_m/T_o = 600/1800 = 0.333 \text{ and so } \operatorname{erf}(x/2\sqrt{kt}) = 0.333$$

$$\text{Therefore, } x/2\sqrt{kt} = -0.304$$

Then x is taken to be –2 because the distance is measured into the country rock, so by solving for t, it is found that $t = 1.082*10^7$ or 125.24 days. This is how long it would be after the initial intrusion of the magma for the country rock 2m from the contact to partially melt.

Cooling of a Lava Lake

The equations that were derived previously in this paper will now be applied to a magma-cooling problem. The first of these cooling problems that will be discussed is the scenario of a cooling lava lake that is in contact with the air. The example discussed here is problem 5-8 from Philpotts.

The given parameters are those of a cooling basaltic lava lake. The initial temperature of the lava is 1200°C. The air temperature, and therefore the upper surface of the lava lake is 25°C. The thermal diffusivity of the basaltic magma and solid crust is $10^{-6} m^2/s$. In this problem, the latent heat of crystallization will be ignored.

In this problem, the air can be thought of in the same way as country rock. Therefore, equation (5) can be used to solve it. The contact temperature or the surface of the lava lake remains constant at 25°C. The key to solving this problem is to adjust the temperatures correctly. We will find the temperature gradients in the upper 1m of this lake at one-hour intervals for each of the first five hours of cooling.

The initial temperature T_0 is taken to be 2350°C, which is 1150°C higher than the actual temperature. The temperature T then at any time t, and depth z, is given as

$$T = T_0 [\ ½ + ½ \text{ erf } [z/(2\sqrt{k*3600})]] \qquad (16)$$

The resulting gradients change as each hour passes.

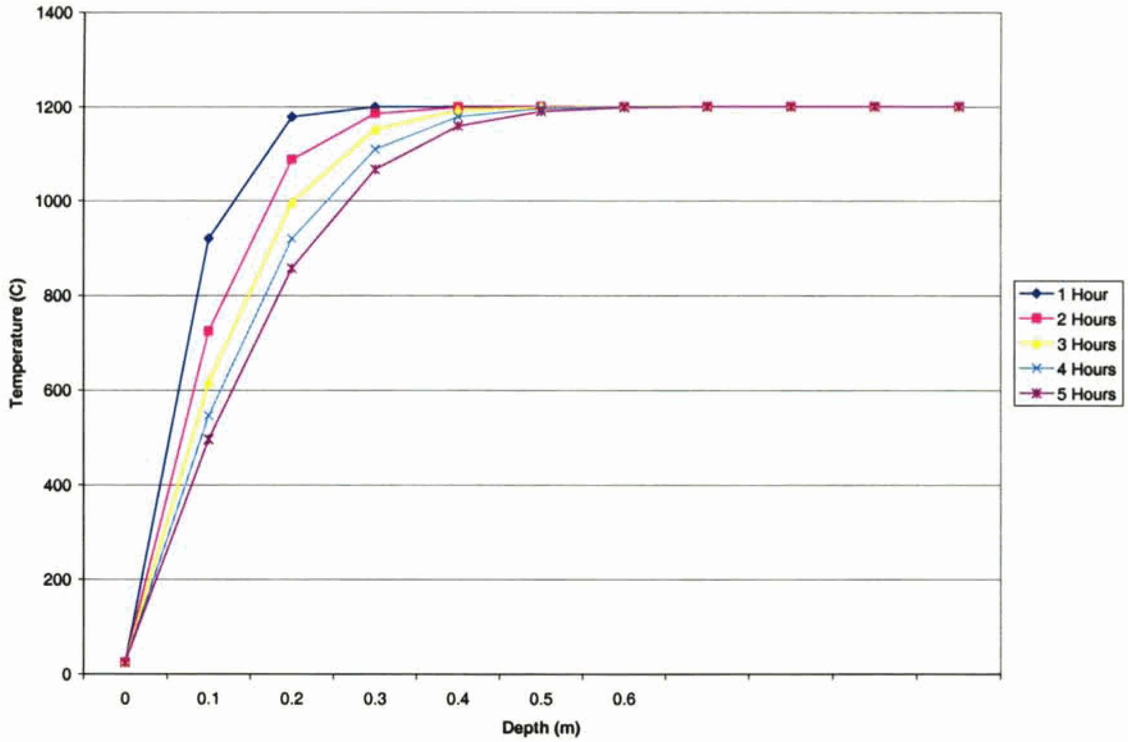

figure 2

It is shown in figure 2 how the temperature increases at a given depth with time at intervals of one hour. An example of the spreadsheet used is given in appendix B.

Cooling of a Lava Lake Numerical Method

This same problem can also be solved using the numerical method. Here, just as described above, the lava is divided into cells in the direction of increasing depth. In this example, the parameters are the same as those used in the previous example. However, this time the calculations will be made for time intervals on the hour up to eight hours.

This problem is best solved using a spreadsheet due to the nature of the math. The temperature of each cell at a given time is affected by the cells adjacent to it. Initially, at T=0, the cells are set up so that cell 1, which is the cell that is taken to be immediately above the surface of the lake, is at 25°C and the rest of the cells are 1200°C.

As time increases, the cells inside the lava will decrease, but cell one remains at 25°C because it is above the surface.

For conservation of energy, the cells must follow equation (8). In this example though, the latent heat of crystallization is ignored so the change in heat in any cell is the amount gained from below minus the amount lost through the top. The temperature change is the amount of heat flow divided by the heat capacity. So then the temperature for any cell is given for the ith cell as

$$T_i = T_i + \{[(k*t)/(z^2)]*[\, T_{i+1} - 2\,T_i + T_{i-1}]\} \qquad (17)$$

The temperature distribution with depth turns out to be very similar to the previous example. The results are shown here in figure 3. With an increased number of cells, the numerical analysis will approach the exact solution. An example of the spreadsheet used is given in appendix C.

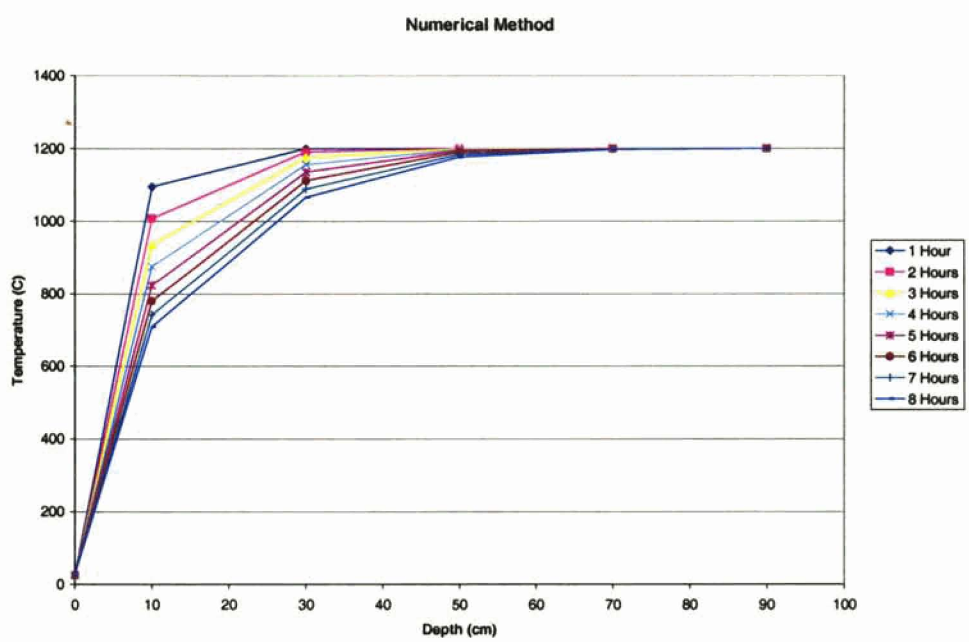

figure 3

Cooling of a Lava Lake Crank-Nicolson Finite Difference method

Here, the same problem of the cooling lava lake is solved using the finite difference method. The time difference is selected to be a half-hour. Therefore, the depth increments that must be used are 0.0424 m so that the equation $kdt/(dz)^2 = 1$ is met. When the calculations are done, a graph is generated of temperatures versus depth in meters. The temperature at the contact will be 25°C for all times because the contact is with the air. At time t=0, the temperatures will be 1200°C at every point but the contact because this is the initial temperature of the magma. As time increases, the temperatures will drop and temperatures further and further from the contact will begin to drop. An example of the spreadsheet used is given in appendix D. The results are shown in figure 4.

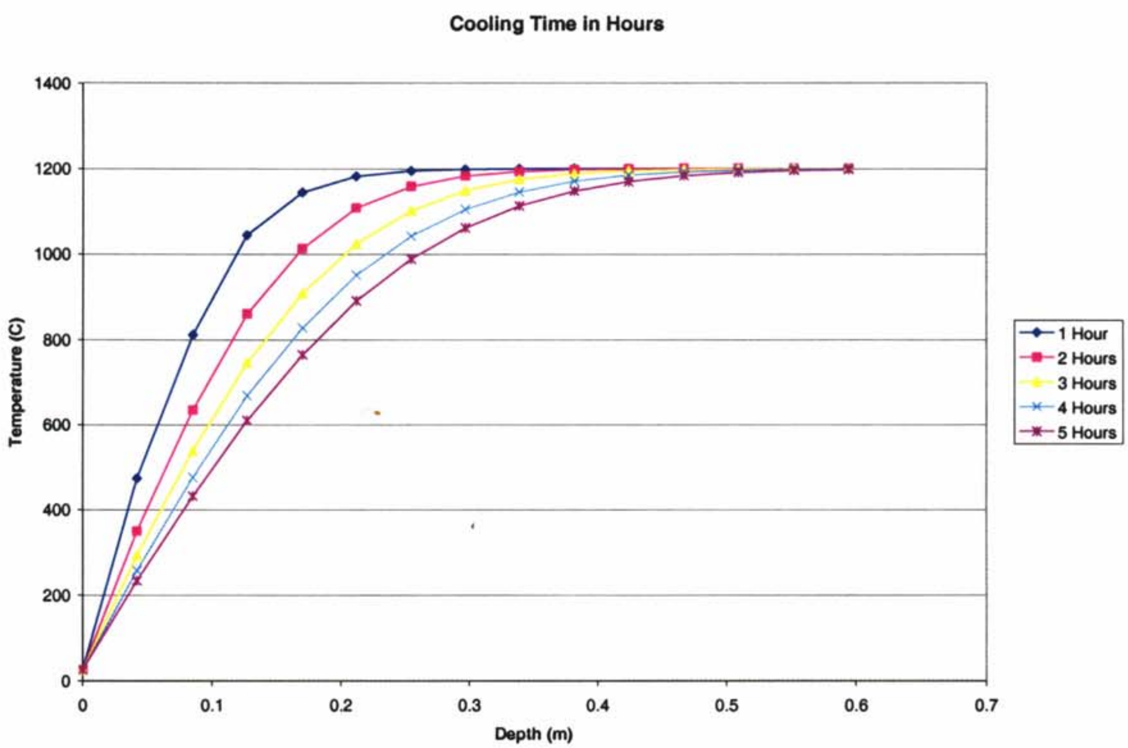

figure 4

Partial Melting Calculation Finite Difference Method

Now that a spreadsheet has been established for using the finite difference method, the problem of partial melting that was discussed above can be solved in the same way. Now the temperatures of the rocks at the contact are 1000°C rather than 25°C that was used in the previous example. At time t=0, every point but the contact will be at 100°C because this is the initial temperature of the country rock. In this example, the temperature is increasing in the country rock with distance from the contact and time since magma emplacement, rather than decreasing in the magma. However, the method holds for both.

Also, notice that at a distance of 2m, the temperature is 703°C at 127 days. This is consistent with the results of 700°C at 125.24 days that were obtained by calculating this time above. An example of the spreadsheet with results is given in appendix E.

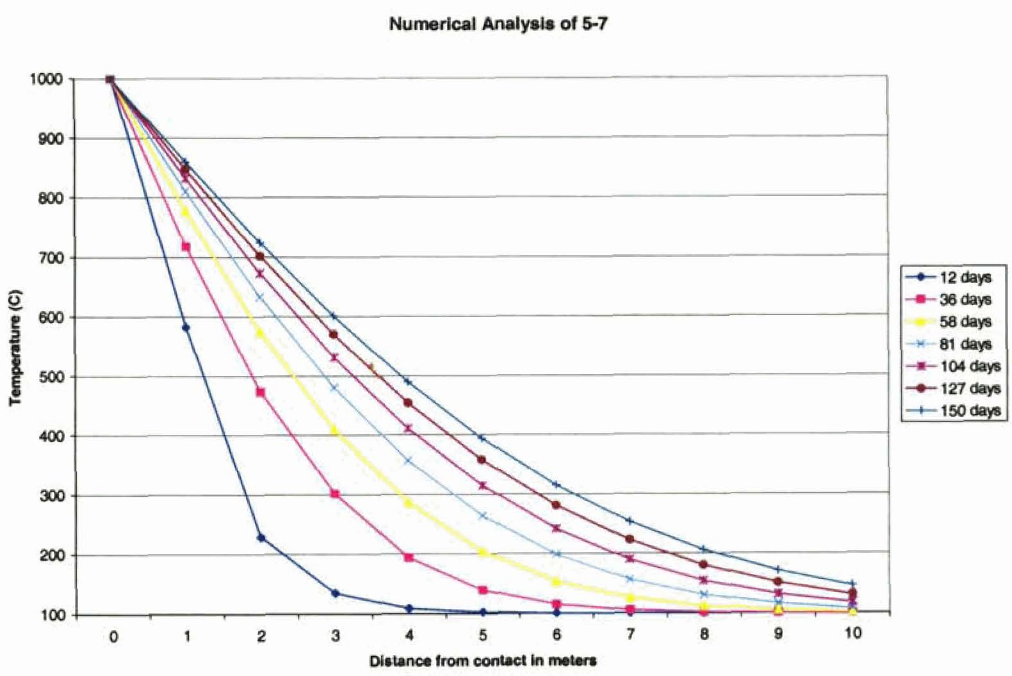

figure 5

Results and Discussion

At this point, a number of methods have been discussed and shown to work in several examples. The true test and purpose of any geologic problem is to apply it to physical circumstances in the real world. Due to the nature of geology, there are usually a number of ways to approach solving a certain problem.

In the following example, the previously discussed methods will be applied to the Egersund Dikes in southwest Norway. By applying these calculations we can hope to know more about the nature of the emplacement of these dikes, or at least solidify ideas about the dikes that are the results of different lines of study.

The Egersund Dikes are emplaced in a swarm, cutting Precambrian country rock along N70W trending faults. The age of the dikes is 630-650 ma as determined by isotope dating. Field data suggests that the fault zone is associated with rifting that occurred prior to the opening of the Iapetus Ocean. Many olivine crystals and glassy margins are present in the dikes (Barton and Miller 1991).

Because the conditions of a cooling dike are not too complicated, the diffusion equation can be used to find exact results. In describing the conditions as not complicated, it is meant that the boundary conditions can be easily determined, and the geometry of the situation is simple and straightforward.

As in any of the applications that have been shown above, the first step that needs to be taken when approaching this problem is to set up some boundary conditions. One of the most important parameters that needs to be known for these calculations is the initial temperature of the magma T_0. Due to studies of the petrology and composition of the dikes, it can be assumed that the magma was about 1200°C at the time of intrusion. This is a very reasonable assumption for magma temperature, and is consistent with the temperature that has been used in the previous examples.

Furthermore, it is observed in thin sections of rocks from this location that there is glassy texture present near the contacts of the dike and the country rock. This is evidence that the dikes were very near or at the surface of the Earth at the time of their emplacement. The dikes were, in other words, not inserted at depth and then later brought to the surface. If this had been the case then the cooling that would have taken place at the edges of the dike would have been slower because of the warmer country rocks at depth that are a result of the geothermal gradient that was discussed earlier. Due to this, it can be assumed that the temperature of the country rocks was close enough to 0°C so that no adjustments need to be made to the temperatures as was done in the first problem above on partial melting.

The thermal diffusivity k, is taken to be $10^{-6} m^2/s$. This is a standard value, that can safely be used for most all rocks and magmas. It is used in most texts as well (Carslaw and Jaeger, Turcotte and Schubert, Philpotts).

The result that is being looked for in these applications is the time t that these dikes took to solidify. The temperatures at which the magma reached solidification have been obtained previously by Dr. Michael Barton. For the majority of the following examples this temperature is taken to be 1000°C. This equation solves for T/T_0. Because we want to take this relationship to the point of solidification, T here will be 1000°C and as mentioned above, T_0 will be 1200°C. Then, the value on the left side of equation (6) will be 0.8333.

The only variables left to assign in this equation are a and x. These are both measures of distance. The first, a is the half width of the dike. The second, x is the distance from the center of the dike measured, starting at the center and increasing outward.

The graphs that will be shown for these cooling relationships will be plotted with T/T_0 on the y-axis and x/a on the x-axis. Therefore, where x/a is equal to 1, this is the edge of the dike where the contact exists with the country rock.

Now that each variable has been assigned, the way that the calculations were done was by taking a time t in seconds and solving the equation for T/T_0 with an incremental list of x values to a reasonable distance from the dike. This produces a curve with the highest temperature ratio at the center of the dike and decreasing outward. This step is repeated with different time values until the time is found where the maximum temperature ratio at the middle of the dike is 0.8333, or in other words the temperature T at the center is 1000^{o}C and the entire dike is solid. This time t then, is the amount of time that it took for this dike to solidify from the time of emplacement.

It is important to note that in these calculations, as in the previous examples, the latent heat of crystallization is not included. This is the heat that is produced within the dike while solidification is taking place and crystals form. If this latent heat were to be included, the time of solidification would be slowed slightly, but not significantly.

Measurements were made in the field of the widths of these dikes. The widths ranged from 2 ft for the thinnest dike to 9 m for the thickest dike. These widths are divided by two to give the value of a. The calculations described here were done for a variety of dike widths within this range. Fairly small increments of x were used so as to get as smooth a curve as possible, producing a rather large number of data points for each curve. Due to the large number of calculations done for each plot, it is necessary to use some sort of spreadsheet or computational software. Here Matlab was used to carry out the calculations and plotting.

The first plot, figure 6, was done for a dike of width 2 ft. The value of a then is half of that and must be converted to meters. As shown here, the time that was needed for the center of the dike to reach 1000°C was only 10 hours.

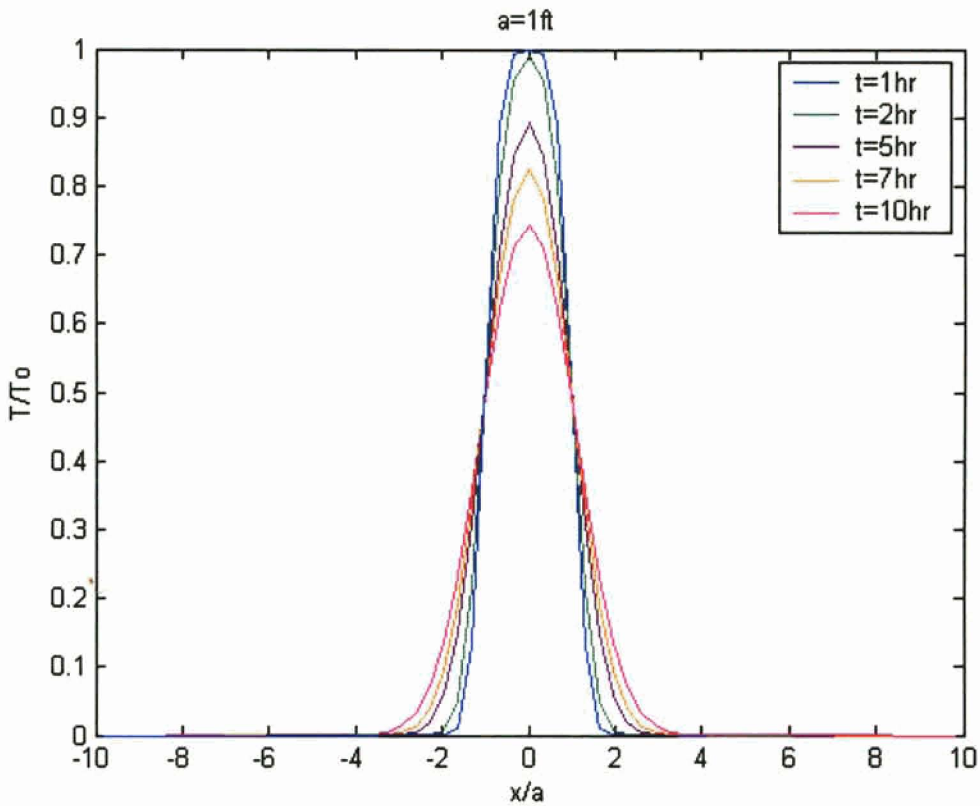

figure 6

The next plot, figure 7, was done for a bit larger dike where the half-width was 1 m. By solving the equation for increasing values of t, it was found that the dike of this size solidified after 80 hours or 3 1/3 days. For this dike the plot is shown only going in one direction from the center outward. It is assumed for all dikes that these properties would be the same throughout, and therefore, the graph would be symmetrical on the other side. It is not necessary to plot both sides of the dike in order to know when the center reaches a temperature of solidification.

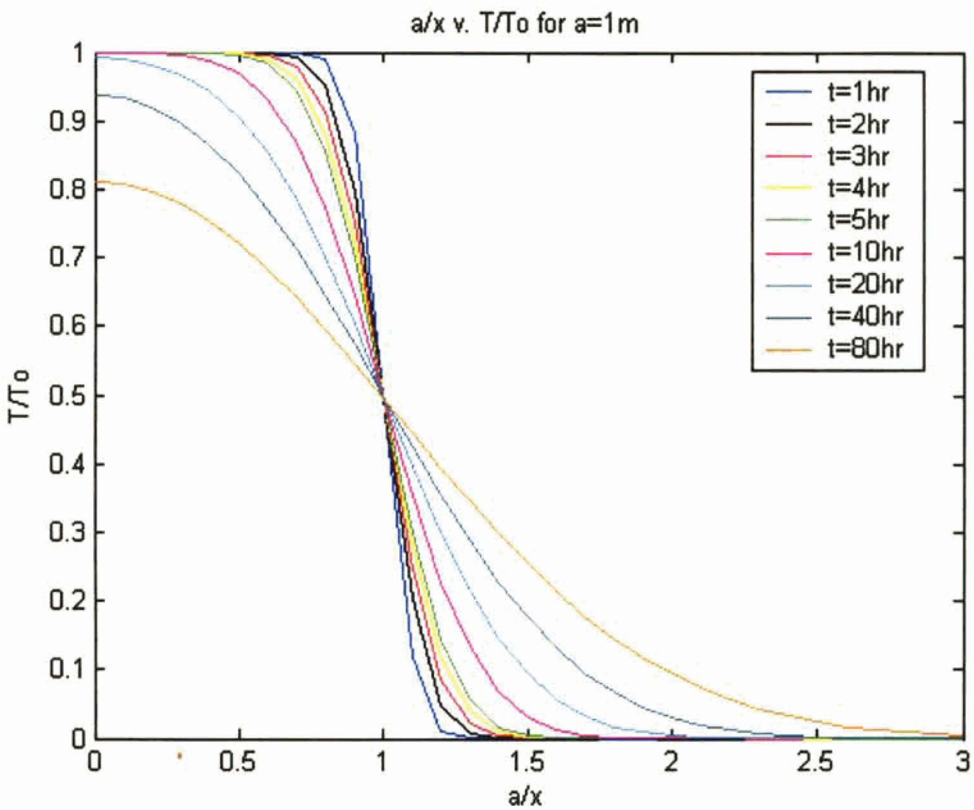

figure 7

Next, another larger dike is considered in figure 8. This dike has a half-width of 2 m. Now, the solidification time has increased to 300 hours, which is 12.5 days and still pretty fast. This plot is also shown only for half of the dike. Note that in each case, the temperature at the contact (*x*/a) remains constant throughout solidification at about 600°C.

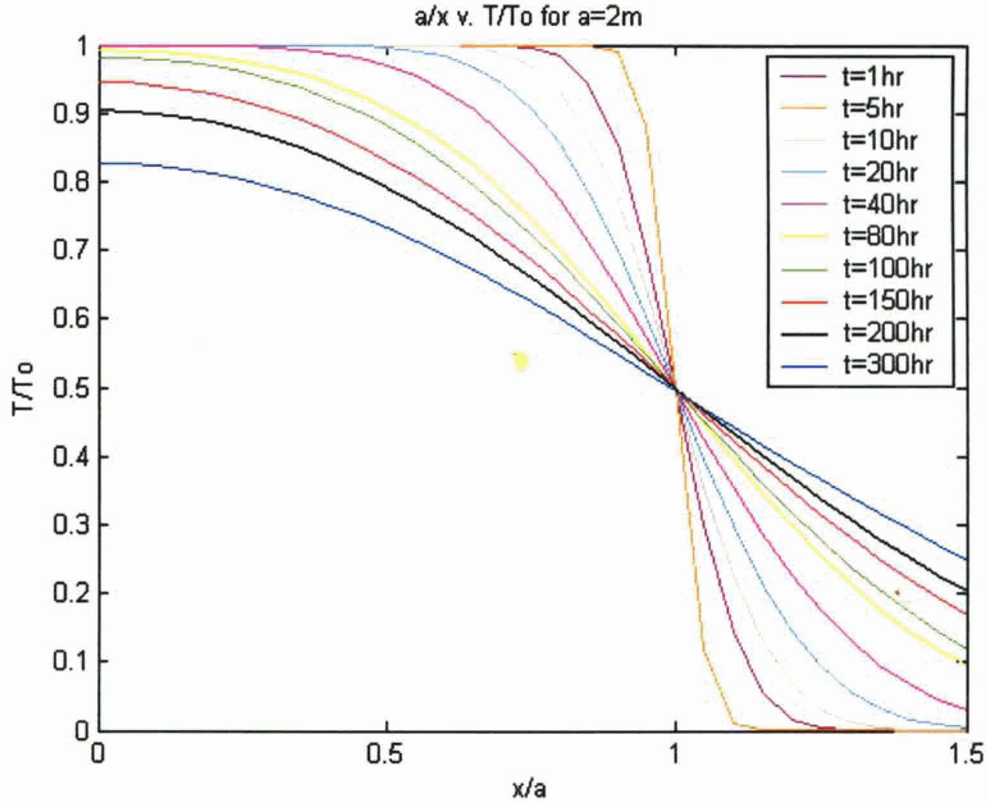

figure 8

The next graph, which is figure 9, shows the calculations for the largest dike that was measured in the area. This dike was 9 meters across. As shown here, the time needed for this dike to solidify is 1500 hours or 62.5 days.

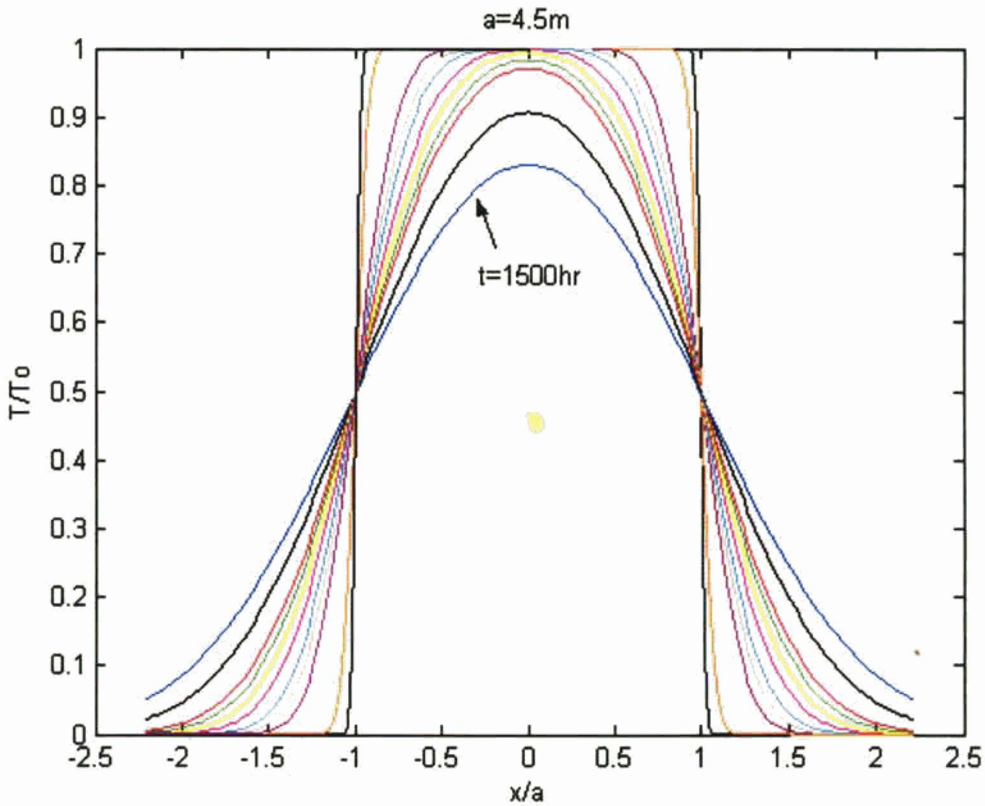

figure 9

This result is a bit suspect. The time that has been arrived at seems a bit long when looking at thin sections of this dike. The textures present in this dike suggest that it cooled more rapidly than this. Calculations were done again for the dike of this width with the parameters changed a bit. It is possible that the initial temperature of the magma was lower than 1200°C, and that the temperature of solidification could have also been lower as well. This set of calculations was done with the initial temperature at 1100°C, and the solidification temperature at 950°C. The T/T_0 value to stop at is now 0.86363, so the time that was determined was a bit shorter at 1300 hours.

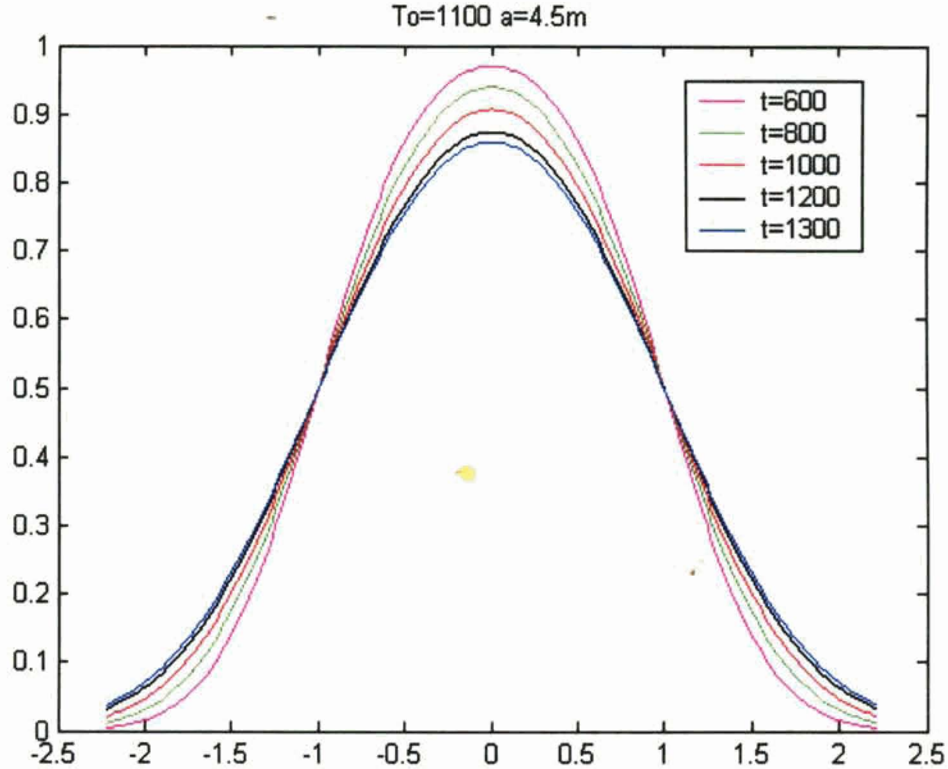

figure 10

These values for solidification times reflect the history of these dikes and are consistent with other findings. The times that have been determined here are all quite short. Even the time for the large dike is still relatively fast for solidification of a dike of this size. These rapid cooling times support the idea that these dikes were emplaced very near if not at the Earth's surface. These calculated cooling times and the glassy textures present in the rocks agree with each other on this point.

In addition to these observed textures, the data is consistent with calculations of a different kind. Dr. Michael Barton has made calculations to determine the time that these dikes took to solidify as well, but went about it in a different manner. These dikes contain olivine crystals. The growth rate of these crystals is known to be 10^{-6} cm/sec^2.

With this growth rate, and the measured diameters of crystals from these dikes which have been obtained from thin sections, the time that these crystals took to grow can easily be calculated. These times were obtained from crystals near the centers of dikes of the sizes used here. The results were within the same order of magnitude with the time measured in seconds and in some cases were quite close. Correlation of the results from these two different methods shows that the both of them are reasonable approaches to this problem and can be used to check each other.

Conclusions

It has been found that while these calculations are fairly simple in practice, it is very important to understand why they are being used and how they have been derived. The procedures are of no use if the boundary conditions are not correct. One cannot correctly determine the boundary conditions without understanding the equations being used. Also, the differences between the methods described must be understood in order to choose the correct one to use.

Another benefit of understanding the methods presented in this paper is the ability to be better prepared in the field. If some of these calculations have been run through prior to a trip in the field to observe a dike system, the geologist can have some idea as to what observations might be expected. In other words when observing a dike if one were to know roughly what temperatures the different parts of the dike were at while it was forming and approximately how long it took to form, then the observer would be more informed as to what type of textures, mineral compositions and other properties to expect.

It can seem like a daunting task to take on calculations of heat conduction. It may seem like too much trouble to go through just to shed a little light on another subject. However, once the basic principles are mastered, calculations of heat conduction by diffusion can be found to be useful. The applications lie in a wide variety of different problems. Also, because of the many applications of the diffusion equation itself, it can be an extremely useful tool for a range of fields of study.

Appendix A

Geothermal Gradient

Feet	Degrees (C)
1	17.90837
2	34.98008
3	51.21514
4	66.61355
5	81.1753
6	94.9004
7	107.7888
8	119.8406
9	131.0558
10	141.4343
11	151.3427
12	161.1474
13	170.8486
14	180.4463
15	189.9403
16	199.3307
17	208.6176
18	217.8008
19	226.8805
20	235.8566
21	244.7291
22	253.498
23	262.1634
24	270.7251
25	279.1833
26	287.5379
27	295.7889
28	303.9363
29	311.9801
30	319.9204
31	327.757
32	335.4901
33	343.1196
34	350.6455
35	358.0678
36	365.3865
37	372.6016
38	379.7132
39	386.7212
40	393.6255
41	398.6996
42	403.7743
43	408.8489
44	413.9235

45	418.9981
46	424.0728
47	429.1474
48	434.222
49	439.2966
50	444.3713
51	449.4459
52	454.5205
53	459.5951
54	464.6698
55	469.7444
56	474.819
57	479.8937
58	484.9683
59	490.0429
60	495.1175
61	500.1922
62	505.2668
63	510.3414
64	515.416
65	520.4907
66	525.5653
67	530.6399
68	535.7146
69	540.7892
70	545.8638
71	550.9384
72	556.0131
73	561.0877
74	566.1623
75	571.2369
76	576.3116
77	581.3862
78	586.4608
79	591.5354
80	596.6101
81	601.6847
82	606.7593
83	611.834
84	616.9086
85	621.9832
86	627.0578
87	632.1325
88	637.2071
89	642.2817
90	647.3563
91	652.431
92	657.5056

93	662.5802
94	667.6549
95	672.7295
96	677.8041
97	682.8787
98	687.9534
99	693.028
100	698.1026

Appendix B

5-8

z (m)	y (1hr)	t (1hr)	erf(y)	T (1hr)	y (2hr)	t (2hr)	erf(y)	T (2hr)
0	0	1	-8E-06	24.9906	0	1	-8E-06	24.9906
0.1	0.833333	0.718361	0.761427	919.6769	0.589256	0.782946	0.59534	724.5251
0.2	1.666667	0.560501	0.981556	1178.329	1.178511	0.643313	0.904419	1087.693
0.3	2.5	0.459522	0.99959	1199.519	1.767767	0.545946	0.987561	1185.384
0.4	3.333333	0.389373	0.999998	1199.997	2.357023	0.474179	0.999137	1198.986
0.5	4.166667	0.337805	1	1200	2.946278	0.419088	0.999969	1199.963
0.6	5	0.298298	1	1200	3.535534	0.375465	0.999999	1199.999
0.7	5.833333	0.267065	1	1200	4.12479	0.340068	1	1200
0.8	6.666667	0.241752	1	1200	4.714045	0.31077	1	1200
0.9	7.5	0.220822	1	1200	5.303301	0.286119	1	1200
1	8.333333	0.203228	1	1200	5.892557	0.265092	1	1200

y (3hr)	t (3hr)	erf(y)	T (3hr)	y (4hr)	t (4hr)	erf(y)	T (4hr)	y (5hr)	t (5hr)	erf(y)	T (5hr)
0	1	-8E-06	24.9906	0	1	-8E-06	24.9906	0	1	-8E-06	24.9906
0.481125	0.815425	0.50374	616.8939	0.416667	0.8361	0.444286	547.036	0.372678	0.850822	0.401813	497.1298
0.96225	0.688369	0.826449	996.0779	0.833333	0.718361	0.761427	919.6769	0.745356	0.740375	0.708175	857.1059
1.443376	0.59557	0.958755	1151.537	1.25	0.629688	0.922894	1109.4	1.118034	0.655307	0.886159	1066.237
1.924501	0.524819	0.993489	1192.349	1.666667	0.560501	0.981556	1178.329	1.490712	0.587774	0.964965	1158.834
2.405626	0.469093	0.999327	1199.209	2.083333	0.505013	0.996773	1196.208	1.86339	0.532859	0.991574	1190.1
2.886751	0.424065	0.999955	1199.947	2.5	0.459522	0.99959	1199.519	2.236068	0.487329	0.998427	1198.152
3.367877	0.386924	0.999998	1199.998	2.916667	0.421549	0.999962	1199.956	2.608746	0.448967	0.999773	1199.734
3.849002	0.355766	1	1200	3.333333	0.389373	0.999998	1199.997	2.981424	0.416204	0.999975	1199.97
4.330127	0.329251	1	1200	3.75	0.36176	1	1200	3.354102	0.387897	0.999998	1199.997
4.811252	0.306415	1	1200	4.166667	0.337805	1	1200	3.72678	0.363195	1	1200

Appendix C

5-9

cell #	Depth(cm)	0hr	1hr	2hr	3hr	4hr	5hr	6hr	7hr	8hr
1	0	25	25	25	25	25	25	25	25	25
2	10	1200	1094.25	1007.535	935.5721	875.1578	823.8763	779.8908	741.7927	708.4924
3	30	1200	1200	1190.483	1174.874	1155.521	1134.025	1111.469	1088.583	1065.845
4	50	1200	1200	1200	1199.143	1197.036	1193.56	1188.751	1182.732	1175.658
5	70	1200	1200	1200	1200	1199.923	1199.67	1199.149	1198.287	1197.032
6	90	1200	1200	1200	1200	1200	1199.993	1199.965	1199.894	1199.759
7	110	1200	1200	1200	1200	1200	1200	1199.999	1199.996	1199.987
8	130	1200	1200	1200	1200	1200	1200	1200	1200	1200
9	150	1200	1200	1200	1200	1200	1200	1200	1200	1200
10	170	1200	1200	1200	1200	1200	1200	1200	1200	1200
11	190	1200	1200	1200	1200	1200	1200	1200	1200	1200

Appendix D

5-10

Depth (m)	0h	.5h	1.0h	1.5h	2.0h	2.5h	3.0h	3.5h	4.0h	4.5h	5.0h
0	25	25	25	25	25	25	25	25	25	25	25
0.042	1200	570.3194	472.9075	392.8965	348.8252	315.9753	291.7441	272.5547	256.9967	244.0233	233.0026
0.085	1200	1031.278	810.3523	711.2337	634.0671	579.8342	537.1423	503.0766	474.9103	451.1828	430.8276
0.127	1200	1154.791	1043.391	935.7397	858.8072	795.729	745.1207	702.8868	667.2028	636.5082	609.7765
0.17	1200	1187.886	1144.049	1077.324	1012.604	956.4102	907.0963	864.2322	826.5922	793.3476	763.7478
0.212	1200	1196.754	1181.26	1148.903	1106.967	1064.137	1023.398	985.5227	950.7564	918.9232	889.7835
0.255	1200	1199.13	1193.974	1180.265	1157.676	1129.858	1100.23	1070.533	1041.668	1014.084	987.9542
0.297	1200	1199.767	1198.116	1192.782	1182.053	1166.274	1147.109	1126.101	1104.293	1082.365	1060.745
0.339	1200	1199.938	1199.423	1197.466	1192.803	1184.758	1173.589	1160.054	1144.916	1128.792	1112.149
0.382	1200	1199.983	1199.826	1199.138	1197.241	1193.465	1187.51	1179.496	1169.774	1158.737	1146.748
0.424	1200	1199.996	1199.948	1199.714	1198.98	1197.32	1194.372	1189.97	1184.154	1177.085	1168.967
0.467	1200	1199.999	1199.985	1199.907	1199.634	1198.942	1197.569	1195.304	1192.043	1187.785	1182.597
0.509	1200	1200	1199.996	1199.97	1199.872	1199.595	1198.988	1197.887	1196.162	1193.738	1190.598
0.552	1200	1200	1199.999	1199.991	1199.956	1199.849	1199.592	1199.083	1198.216	1196.908	1195.104
0.594	1200	1200	1200	1199.997	1199.985	1199.945	1199.841	1199.615	1199.201	1198.531	1197.549
0.636	1200	1200	1200	1199.999	1199.995	1199.981	1199.94	1199.846	1199.66	1199.34	1198.844
0.679	1200	1200	1200	1200	1199.999	1199.994	1199.98	1199.946	1199.876	1199.75	1199.546
0.721	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200

Appendix E

5-11

Dist. (m)	0 days	12 days	23 days	35 days	46 days	58days	69 days
0	1000	1000	1000	1000	1000	1000	1000
1	100	582.3085	656.9219	718.2069	751.9636	777.1252	795.6851
2	100	229.2342	398.4536	474.3741	533.4804	575.0203	607.7202
3	100	134.6282	219.9556	302.4121	361.3389	409.6536	448.4167
4	100	109.2786	142.856	193.9648	243.5363	286.5775	324.3489
5	100	102.4862	114.354	139.1374	171.2567	204.061	235.2644
6	100	100.6662	104.6151	115.1136	132.412	153.7182	176.4129
7	100	100.1784	101.4418	105.5214	113.7325	125.8226	140.5036
8	100	100.0476	100.4384	101.9185	105.4861	111.6741	120.2093
9	100	100.0119	100.1215	100.5892	102.1011	105.04	109.4708
10	100	100	100	100	101	102	104

81 days	93 days	104 days	116 days	127 days	139 days	150 days
1000	1000	1000	1000	1000	1000	1000
810.3832	822.2998	832.2367	840.6779	847.9659	854.34	859.9767
633.8127	655.3865	673.5604	689.1513	702.7121	714.6481	725.2589
480.7655	508.0974	531.6077	552.083	570.1215	586.1651	600.5537
357.1804	386.0102	411.4736	434.1468	454.4758	472.8243	489.4833
264.2748	290.903	315.2863	337.61	358.0886	376.9222	394.2923
199.1571	221.2643	242.3971	262.4226	281.3091	299.0794	315.782
156.5857	173.2937	190.1052	206.689	222.849	238.4579	253.4559
130.5633	142.1902	154.5693	167.3671	180.297	193.1459	205.8163
115.6931	123.1884	131.6899	140.9841	150.666	160.6107	170.7405
108	112	118	124	131	138	146

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